Prepositioning of Emergency Supplies for Predictable Disasters Using Distributionally Robust Optimization

Jing Li, Ada Che, Feng Chu

Preprints, 10th IFAC Conference on
Manufacturing Modelling, Management and Control
June 22-24, 2022. Nantes, France

Prepositioning of Emergency Supplies for Predictable Disasters Using Distributionally Robust Optimization

Jing Li*, **, Ada Che*, Feng Chu**

** School of Management, Northwestern Polytechnical University, Xi'an, China;*
{e-mail: lijing2020@mail.nwpu.edu.cn, ache@nwpu.edu.cn}

*** IBISC, Univ Évry, University of Paris-Saclay, Évry, France*
{e-mail: feng.chu@univ-evry.fr}.

Abstract: This paper examines the prepositioning of emergency supplies problem, which integrates the decisions of facility location, emergency supplies prepositioning, and distribution under predictable disasters. We scrutinize three stages of relief management in the context of predictable disasters. After that, this paper introduces a novel three-stage distributionally robust optimization (3DRO) model. To make the 3DRO model computationally tractable, we further develop a deterministic equivalent model. A real case study in China demonstrates the superiority of our proposed model.

Keywords: disaster relief; inventory prepositioning; facility location; predictable disasters; distributionally robust optimization

1. INTRODUCTION

In recent years, with the growth of population, the development of urbanization, and the change of climate, the impact of natural disasters on people has become more and more severe (Aleksandrova et al., 2021). To facilitate the response when disasters occur, such as earthquakes and typhoons, disaster managers usually make emergency supplies prepositioning decisions in advance.

However, the characteristics of typhoons and earthquakes are inherently different. The former is predictable and repetitive, while the latter is rapid-onset and unforeseen. Unlike unpredictable disasters, predictable disasters can be observed a few days before their occurrence, and disaster warnings will be issued at the right time. Therefore, the preparedness phase of predictable disaster can be divided into two stages: before disaster warning and disaster warning to disaster occurrence. However, if we only consider the period before a disaster but ignore the phase when the disaster occurs, sub-optimal solutions may be obtained (Ni et al., 2018). Thus, it is necessary to study the emergency supplies prepositioning problem by comprehensively considering the three stages of before disaster warning, disaster warning to disaster occurrence, and disaster response.

Uncertainty is another essential feature of natural disasters. Specifically, the scope and intensity of a disaster are usually uncertain before it is revealed. In addition, since natural disasters are high-impact, low-probability (HILP) events, it is difficult to assess the impact of future disaster events and predict the emergency supplies demands under these events. To address this issue, many researchers built 2-stage stochastic optimization (2SO) models to handle the uncertainty for the prepositioning of emergency supplies problems in the context of predictable disasters. They usually examined the latter two stages: disaster warning to disaster occurrence and disaster response (Pacheco and Batta, 2016, Paul and Zhang, 2019, Rezapour et al., 2021, Stauffer and Kumar, 2021).

Nevertheless, there are two challenges to the 2SO models these researchers developed. First of all, the 2SO models they built are risk-neutral, which is inconsistent with the risk attitude of disaster managers. Moreover, the probability of disaster scenarios is arduous to estimate accurately because of the low frequency of disasters. Thus, some researchers favored robust optimization (RO) because it does not require too much information and results in risk-averse decisions (Wang and Paul, 2020, Dalal and Üster, 2021).

Because the RO models can resist any disturbance in the uncertainty set and rarely utilize historical data that exist in reality, the solutions of the RO models tend to be overconservative. In recent years, some researchers have utilized the distributionally robust optimization (DRO) approach to model the prepositioning of emergency supplies problems in the context of disaster management (Liu et al., 2019, Wang et al., 2021). DRO is a modeling technique that assumes only partial distributional information, finding optimal decisions under the worst-case probability measures in the ambiguity set. To the best of our knowledge, no scholars have integrated the three stages of predictable disasters and employed the DRO technique to model this problem. This paper fills the gap in research.

We explore novel ways to investigate the prepositioning of emergency supplies problem, which integrates facility location, relief supplies prepositioning, and distribution decisions under predictable disasters. The main contributions of our study can be summarized as follows.

First, to the best of our knowledge, this paper is the first to consider three stages of predictable disasters comprehensively, involving the stages of before disaster warning, disaster warning to disaster occurrence, and disaster response. We aim to minimize the total social cost while considering the equality of disaster relief.

Second, a novel 3-stage distributionally robust optimization model (3DRO) for prepositioning the emergency supplies problem is proposed. Specifically, we adopt the scenario generation approach to deal with the uncertainty of the scope and intensity of the disaster. The uncertainties of transportation cost, demand, and the available proportion of a facility in each disaster scenario are modeled by robust optimization. Furthermore, the probability of a disaster scenario is also uncertain, and we adopt DRO to express it.

Third, a tractable counterpart is introduced to solve the 3DRO model. The proposed 3DRO model is highly nonlinear. More specifically, the 3DRO model is multi-stage and multi-level. To make the 3DRO model computationally tractable, we present the reformulation of our proposed model.

The remainder of this paper is organized as follows. The 3DRO model is introduced in Section 2. Section 3 provides the reformulation of our proposed model. Moreover, the superiority of the 3DRO is evaluated using a real case in Section 4. Finally, Section 5 concludes this study.

2. PROBLEM DESCRIPTION AND MODEL FORMULATION

2.1 Problem description

We consider building a three-tier emergency network consisting of Major Distribution Centers (MDCs), Pre-staging Areas (PSAs), and Demand Points (DPs). The MDCs are strategic and permanent distribution centers that maintain inventory for disasters. PSAs are used to stockpile supplies in response to pre-disaster warnings temporarily. The DPs are staging areas or points of distribution where supplies are further arranged and delivered to beneficiaries when a disaster occurs (Stauffer and Kumar, 2021).

The decisions in the first stage are strategic decisions, including determining the optimal locations of MDCs, the amount of relief supplies stored in MDCs, and the candidate list of PSA. Although the PSA is a temporary storage facility, to quickly put the PSA into operation after disaster warning, disaster managers require to carry out certain modifications to some facilities, such as stadiums, and schools, in this stage to make them potential PSAs. When a disaster warning issues, to improve the response efficiency after the disaster occurs, disaster managers will transport the materials in MDCs to PSAs in advance, which are closer to DPs. The closer PSAs can help in getting the relief supplies to the victims on time. However, the closer the PSAs to DPs, the greater the risk of being damaged by a disaster, making them partially unavailable. Thus, the tactical decisions in the second stage are the amount of emergency materials transferred from MDCs to optimal PSAs. Finally, after a disaster, the disaster managers optimize the amount of supplies shipped from PSAs to DPs to meet the requirements of the affected people. However, the materials in the PSAs may not be sufficient to satisfy the victims, so it is necessary to decide the amount of materials shipped from MDCs to DPs directly.

It is worth noting that there are various uncertainties in making decisions on this problem. First of all, the damage caused by a typhoon is closely related to typhoon intensity and typhoon tracks. According to the classification of the tropical cyclone, typhoon intensity can be mainly divided into five categories: tropical storm (TS), severe tropical storm (STS), typhoon (TY), strong typhoon (STY), and super typhoon (SuperTY). This paper uses disaster scenarios to reflect the uncertainty of both the typhoon track and its intensity. However, even if the typhoon intensity and typhoon track are determined, the damage remains uncertain specified for a scenario, including the supplies demands, travel time of roads, and the available proportion of PSAs after a disaster. Furthermore, the probability of disaster scenarios is also uncertain.

2.2 Model formulation

Before introducing the proposed model, we present the following primary notations used in this paper.

Sets

I	The set of Major Distribution Centers (MDCs)
J	The set of Pre-staging Areas (PSAs)
K	The set of Demand Points (DPs)
K_j	The set of DPs that can be covered by PSA j. $K_j = \{k \in K \mid dis(j,k) \leq \tau\}$, where $dis(j,k)$ is the distance from PSA j to DP k, and τ is the distance threshold value
A	The set of capacity levels for MDCs
S	The set of disaster scenarios

Parameters

F_{ia}	Fixed cost of locating and managing an MDC with capacity level a at node i
G_a	The amount of relief supplies that can be stored in an MDC with capacity level a
F_j	Fixed cost of making the facility a potential PSA at node j
G_j	The amount of relief supplies that can be stored in a PSA j
C^z	Handling cost of per unit relief supplies
C_{ij}^f	Transportation cost of per unit relief supplies from MDC i to PSA j
U	Penalty cost for per unit unused relief supplies at PSAs
O^s	Penalty cost for per unit of unsatisfied demand at the first distribution in scenario s
V^s	Penalty cost for per unit of unsatisfied demand at DPs in scenario s
$\tilde{p}^s$	The random probability of a disaster scenario s
$\tilde{C}_{ik}^s$	Random transportation cost of each unit relief supplies from MDC i to DP k in scenario s
$\tilde{C}_{jk}^s$	Random transportation cost of each unit relief supplies from PSA j to DP k in scenario s
$\tilde{V}_j^s$	Random proportion of PSA j that is available in scenario s

$\widetilde{D}_k^s$ Random demands for relief supplies at DP k in scenario s

Decision variables

x_{ia} 1, if an MDC with capacity level a opens at node i and 0 otherwise

y_j 1, if a PSA at node j is on the candidate list and 0 otherwise

z_i The amount of relief supplies stored at MDCs i

f_{ij}^s Flow quantity from MDC i to PSA j in scenario s

g_{ik}^s Flow quantity from MDC i to DP k in scenario s

h_{jk}^s Flow quantity from PSA j to DP k in scenario s

l_j^s The amount of unused relief supplies at PSA j in scenario s

n_k^s The amount of unsatisfied demands at DP k in scenario s

We formulate the proposed 3DRO model as follows.

$$\min\left\{\sum_{i\in I}\sum_{a\in A}F_{ia}x_{ia}+\sum_{j\in J}F_j\,y_j+\sum_{i\in I}C^z z_i\right.$$
$$+\max_{\tilde{p}^s\in\mathbb{P}}\mathbb{E}\left[\min\left(\sum_{i\in I}\sum_{j\in J}C_{ij}^f f_{ij}^s+\sum_{j\in J}U\cdot l_j^s\right.\right.$$
$$+\max_{\tilde{C}_{ik}^s\in\mathbb{C}_1^s,\tilde{C}_{jk}^s\in\mathbb{C}_2^s}\min\sum_{k\in K}V^s n_k^s+\sum_{j\in J}\sum_{k\in K_j}\tilde{C}_{jk}^s h_{jk}^s$$
$$\left.\left.\left.+\sum_{i\in I}\sum_{k\in K}(\tilde{C}_{ik}^s+O^s)g_{ik}^s\right)\right]\right\} \tag{1}$$

s.t.
$$z_i\le\textstyle\sum_{a\in A}G_a x_{ia}\,,\forall i\in I, \tag{2}$$
$$\sum_{a\in A}x_{ia}\le 1,\forall i\in I, \tag{3}$$
$$\sum_{i\in I}f_{ij}^s\le G_j y_j,\forall j\in J,s\in S, \tag{4}$$
$$\sum_{j\in J}f_{ij}^s+\sum_{k\in K}g_{ik}^s\le z_i,\forall i\in I,s\in S, \tag{5}$$
$$\sum_{k\in K_j}h_{jk}^s+l_j^s=\tilde{V}_j^s\sum_{i\in I}f_{ij}^s\,,\forall\tilde{V}_j^s\in\mathbb{V}^s,j\in J,s\in S, \tag{6}$$
$$\sum_{i\in I}g_{ik}^s+\sum_{j\in J_k}h_{jk}^s+n_k^s=\widetilde{D}_k^s,$$
$$\forall\widetilde{D}_k^s\in\mathbb{D}^s,k\in K,s\in S, \tag{7}$$
$$x_{ia},y_j\in\{0,1\},\forall i\in I,a\in A,j\in J, \tag{8}$$
$$z_i\ge 0,\forall i\in I, \tag{9}$$
$$f_{ij}^s\ge 0,\forall i\in I,j\in J,s\in S, \tag{10}$$
$$h_{jk}^s\ge 0,\forall\,j\in J,k\in K_j,s\in S, \tag{11}$$
$$g_{ik}^s,l_j^s,n_k^s\ge 0,\forall i\in I,j\in J,k\in K,s\in S. \tag{12}$$

There are three stages in the objective function (1), aiming to minimize total rescue costs. In the first stage, disaster managers optimize pre-disaster strategic plans to minimize the total costs of opening MDCs, building the candidate list of PSAs, and emergency supplies prepositioning. The second stage contains the tactical decisions after the disaster warning. It minimizes the expected cost of transportation of emergency supplies from MDCs to PSAs, and the expected penalty cost of unused commodities at PDAs under the worst disaster scenario probability distribution. Predictable disasters such as typhoons are cyclical and repetitive, and partial knowledge of scenario probability distribution can be obtained from historical data and expert experience. To make trade-offs between the reliability and conservatism of decisions, we use DRO in this stage. The tactical decisions after the disaster warning directly affect the efficiency and effectiveness of disaster relief. The inadequate deployment will be criticized by people, and it is contrary to the purpose of humanitarian logistics. Given the above reasons, robust optimization is adopted to ensure the reliability of the rescue decisions. Therefore, we minimize the penalty costs of unsatisfied commodities at DPs, and the sum of the transportation costs in the last stage after the worst-case uncertainty unfolds.

Constraints (2) state that relief supplies can only be stockpiled in an open MDC and are subject to capacity limitations. Constraints (3) restrict that, at most, one MDC is built at node i. Constraints (4) specify the upper bound of relief flow quantity to PSAs. Constraints (5) ensure that the relief flow quantity from each MDC to PSAs and DPs cannot exceed the amount of relief stored in the MDC. Constraints (6) and (7) ensure the conservation of supply flow at PSAs and DPs, respectively. Constraints (8) - (12) specify the variable range requirements.

3. REFORMULATION

3.1 *The uncertainties in the proposed model*

Because of the low frequency of disasters, it is difficult to estimate the probability distribution of disaster scenarios accurately. We construct the ambiguity set $\mathbb{P}$ as follows.

$$\mathbb{P}=\left\{\tilde{p}^s\in\mathbb{R}_+\middle|\begin{array}{l}\tilde{p}^s=\bar{p}^s+\hat{p}^s,\forall s\in S,\\ \sum_{s\in S}\hat{p}^s=0\,,\forall s\in S,\\ p_{low}^s\le\hat{p}^s\le p_{up}^s,\forall s\in S.\end{array}\right\} \tag{13}$$

where $\bar{p}^s$ is the nominal probability of disaster scenario s, and the variable value of $\bar{p}^s$ is $\hat{p}^s$, which is limited by p_{low}^s, p_{up}^s, and the constraint $\sum_{s\in S}\hat{p}^s=0$.The ambiguity set $\mathbb{P}$ has the following advantages: First, it is smooth for disaster managers to understand and model; Also, it has better performance than other ambiguity sets, such as polyhedral ambiguity sets (Ma et al., 2020); In addition, the proposed model can be computationally tractable.

Because the uncertainties of $\tilde{C}_{ij}^s$ and $\tilde{C}_{kj}^s$ only appear in the objective function (1), we can utilize the budget uncertainty set to describe their uncertainty (Bertsimas and Sim, 2004), as shown in (14) and (15).

$$\mathbb{C}_1^s=\left\{\tilde{C}_{ik}^s\middle|\begin{array}{l}\tilde{C}_{ik}^s=\bar{C}_{ik}^s+\lambda_{ik}^s\cdot\hat{C}_{ik}^s,\forall i\in I,k\in K,\\ 0\le\lambda_{ik}^s\le 1,\forall i\in I,k\in K,\\ \sum_{i\in I}\sum_{k\in K}\lambda_{ik}^s\le\Gamma^{s,C1}.\end{array}\right\} \tag{14}$$

$$\mathbb{C}_2^s = \left\{ \tilde{C}_{jk}^s \middle| \begin{array}{l} \tilde{C}_{jk}^s = \bar{C}_{jk}^s + v_{jk}^s \cdot \hat{C}_{jk}^s, \forall j \in J, k \in K_j, \\ 0 \le v_{jk}^s \le 1, \forall j \in J, k \in K_j, \\ \sum_{j\in J}\sum_{k\in K_j} v_{jk}^s \le \Gamma^{s,C2}. \end{array} \right\} \quad (15)$$

where the nominal value of $\tilde{C}_{ik}^s$ is $\bar{C}_{ik}^s$, the scaled deviation of $\bar{C}_{ik}^s$ is λ_{ik}^s, and the maximum deviation from its nominal value is $\hat{C}_{ik}^s$. $\Gamma^{s,C1}$ is a parameter called "uncertainty budget". The meaning of parameters and variables in (15) can be referred to (14).

For the uncertain parameters on the right-hand side of constraints, the budget uncertainty set proposed by Bertsimas and Sim (2004) is not applicable because the technique they proposed is more specific to the model with uncertain parameters on the objective function and left-hand side of constraints. Therefore, this paper uses the interval uncertainty set to model uncertainties of $\tilde{V}_j^s$ and $\tilde{D}_k^s$.

$$\mathbb{V}^s = \{\tilde{V}_j^s | \tilde{V}_j^s \in [\bar{V}_j^s - \Gamma^{s,V}\hat{V}_j^s, \bar{V}_j^s + \Gamma^{s,V}\hat{V}_j^s], \forall j \in J\} \quad (16)$$

$$\mathbb{D}^s = \{\tilde{D}_k^s | \tilde{D}_k^s \in [\bar{D}_k^s - \Gamma^{s,D}\hat{D}_k^s, \bar{D}_k^s + \Gamma^{s,D}\hat{D}_k^s], \forall k \in K\} \quad (17)$$

where $\bar{V}_j^s, \bar{D}_k^s, \hat{V}_j^s, \hat{D}_k^s$ and $\Gamma^{s,V}, \Gamma^{s,D}$ have similar meanings to the parameters in (14). Obviously, when the worst case is obtained, we have

$$\tilde{V}_j^s = \bar{V}_j^s - \Gamma^{s,V} \cdot \hat{V}_j^s, \forall j \in J, s \in S,$$
$$\tilde{D}_k^s = \bar{D}_k^s + \Gamma^{s,D} \cdot \hat{D}_k^s, \forall k \in K, s \in S.$$

3.2 *Model reformulation*

The 3DRO model (1) - (12) can be written in an abstract form:

$$\min_{\boldsymbol{u}} \boldsymbol{a^T u} + \max_{\boldsymbol{p^s}\in\mathbb{P}} \min_{\boldsymbol{v^s}} (\boldsymbol{p^s})^{\boldsymbol{T}} \left[\boldsymbol{b^T v^s} + \max_{\boldsymbol{q^s}\in\mathbb{Q}^s} \min_{\boldsymbol{w^s}} (\boldsymbol{q^s} + \boldsymbol{r^s})^T \boldsymbol{w^s}\right] \quad (18)$$

$$\text{s.t.} \quad \boldsymbol{Cu} \le \boldsymbol{d} \quad (19)$$

$$\boldsymbol{Ev^s} \le \boldsymbol{Gu}, \forall s \in S, \quad (20)$$

$$\boldsymbol{H^s v^s} + \boldsymbol{I^s w^s} \le \boldsymbol{Ju}, \forall s \in S, \quad (21)$$

$$\boldsymbol{K^s w^s} = \boldsymbol{L^s v^s} + \boldsymbol{m^s}, \forall s \in S, \quad (22)$$

$$\boldsymbol{u} \in \mathbb{Z}_+^{o_1} \times \mathbb{R}_+^{r_1}, \boldsymbol{v^s} \in \mathbb{R}_+^{r_2}, \boldsymbol{w^s} \in \mathbb{R}_+^{r_3}, \forall s \in S, \quad (23)$$

where $\boldsymbol{u}$ stands for the first stage decision variables x_{ia}, y_j and z_i; $\boldsymbol{v^s}$ represents the second stage decision variable f_{ij}^s and l_j^s; $\boldsymbol{w^s}$ denotes the decision variables g_{ik}^s, h_{jk}^s, and n_j^s in the third stage; $\boldsymbol{p^s} = (\tilde{p}^1, \tilde{p}^2, ..., \tilde{p}^{|S|})^T$, and $\boldsymbol{q^s}$ denotes the uncertainty parameters $\tilde{C}_{ik}^s$ and $\tilde{C}_{jk}^s$; $\mathbb{Q}^s$ stands for $\mathbb{C}_1^s$ and $\mathbb{C}_2^s$. o_1, r_1, r_2, and r_3 are the dimensions of the vector spaces of decision variables. Uppercase bold letters stand for matrices, and lowercase bold letters represent vectors. The superscript *s* indicates that the variables or parameters are scenario-dependent. Constraint (19) corresponds to constraints (2) - (3); constraints (20) correspond to constraints (4); constraints (21) stand for constraints (5); constraints (22) denote constraints (6) - (7), and constraints (23) represent constraints (8) - (12).

Theorem 1. The deterministic equivalent model for the 3DRO model can be formulated as follow.

$$\min_{\boldsymbol{u}} \boldsymbol{a^T u} + (\bar{\boldsymbol{p}}^s)^{\boldsymbol{T}} \left[\boldsymbol{b^T v^s} + (\bar{\boldsymbol{q}}^s + \boldsymbol{r^s})^T \boldsymbol{w^s} - \psi^s + \sum_{i\in I}\sum_{k\in K} \alpha_{ik}^s \right.$$
$$\left. + \sum_{j\in J}\sum_{k\in K_j} \gamma_{jk}^s + \Gamma^{s,C1}\beta^s + \Gamma^{s,C2}\delta^s \right] + \sum_{s\in S} p_{low}^s \varphi^s + \sum_{s\in S} p_{up}^s \phi^s$$

s.t. (19) - (23),

$$\alpha_{ik}^s + \beta^s \ge \hat{C}_{ik}^s g_{ik}^s, \forall i \in I, k \in K, s \in S,$$
$$\gamma_{jk}^s + \delta^s \ge \hat{C}_{jk}^s h_{jk}^s, \forall j \in J, k \in K_j, s \in S,$$
$$\omega + \varphi^s + \phi^s + \psi^s = \boldsymbol{b^T v^s} + (\bar{\boldsymbol{q}}^s + \boldsymbol{r^s})^T \boldsymbol{w^s} + \sum_{i\in I}\sum_{k\in K} \alpha_{ik}^s$$
$$+ \sum_{j\in J}\sum_{k\in K_j} \gamma_{jk}^s + \Gamma^{s,C1}\beta^s + \Gamma^{s,C2}\delta^s, \forall s \in S,$$
$$\beta^s \ge 0, \delta^s \ge 0, \varphi^s \le 0, \phi^s \ge 0, \psi^s \le 0, \forall s \in S,$$
$$\alpha_{ik}^s \ge 0, \forall i \in I, k \in K, s \in S,$$
$$\gamma_{jk}^s \ge 0, \forall j \in J, k \in K_j, s \in S,$$

where $\bar{\boldsymbol{p}}^s$ and $\bar{\boldsymbol{q}}^s$ are the nominal values of $\boldsymbol{p^s}$ and $\boldsymbol{q^s}$, respectively.

Proof: We define the inner max-min problem as $\mathrm{P}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s})$.

$$\mathrm{P}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s}): \max_{\boldsymbol{q^s}\in\mathbb{Q}^s} \min_{\boldsymbol{w^s}} (\boldsymbol{q^s} + \boldsymbol{r^s})^T \boldsymbol{w^s} \quad (24)$$

s.t. (21), (22), and $\boldsymbol{w^s} \in \mathbb{R}_+^{r_3}, \forall s \in S.$

For a given $\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s}, \boldsymbol{q^s}$, the inner problem of $\mathrm{P}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s})$ is a linear programming problem. According to the saddle point theorem (Boyd and Vandenberghe, 2004), equation (24) is equivalent to equation (25).

$$\mathrm{P}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s}): \min_{\boldsymbol{w^s}} \max_{\boldsymbol{q^s}\in\mathbb{Q}^s} (\boldsymbol{q^s} + \boldsymbol{r^s})^T \boldsymbol{w^s} \quad (25)$$

Supposed that α_{ik}^s, β^s and γ_{jk}^s, δ^s are the dual variables of the last two expressions in (14) and (15), respectively. By the strong duality theorem, problem $\mathrm{P}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s})$ is equivalent to problem $\mathrm{D}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s})$ as follows.

$$\min_{\boldsymbol{w^s}} (\bar{\boldsymbol{q}}^s + \boldsymbol{r^s})^T \boldsymbol{w^s} + \sum_{i\in I}\sum_{k\in K} \alpha_{ik}^s + \sum_{j\in J}\sum_{k\in K_j} \gamma_{jk}^s$$
$$+ \Gamma^{s,C1}\beta^s + \Gamma^{s,C2}\delta^s$$

s.t. (21), (22), and $\boldsymbol{w^s} \in \mathbb{R}_+^{r_3}, \forall s \in S,$

$$\alpha_{ik}^s + \beta^s \ge \hat{C}_{ik}^s g_{ik}^s, \alpha_{ik}^s \ge 0, \beta^s \ge 0, \forall i \in I, k \in K, s \in S$$
$$\gamma_{jk}^s + \delta^s \ge \hat{C}_{jk}^s h_{jk}^s, \gamma_{jk}^s \ge 0, \delta^s \ge 0, \forall j \in j, k \in K_j, s \in S.$$

Similarly,

$$\min_{\boldsymbol{u}} \boldsymbol{a^T u} + \max_{\boldsymbol{p^s}\in\mathbb{P}} \min_{\boldsymbol{v^s}} (\boldsymbol{p^s})^{\boldsymbol{T}} [\boldsymbol{b^T v^s} + \mathrm{D}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s})]$$

$$\Leftrightarrow \min_{\boldsymbol{u}} \boldsymbol{a^T u} + \min_{\boldsymbol{v^s}} \max_{\boldsymbol{p^s}\in\mathbb{P}} (\boldsymbol{p^s})^{\boldsymbol{T}} [\boldsymbol{b^T v^s} + \mathrm{D}^s(\boldsymbol{u}, \boldsymbol{p^s}, \boldsymbol{v^s})]$$

Supposed that ω is the dual variable of $\sum_{s\in S} \hat{p}^s = 0$; ψ^s is the dual variable of $\bar{p}^s + \hat{p}^s \ge 0$, and φ^s, ϕ^s are the dual variable of $p_{low}^s \le \hat{p}^s \le p_{up}^s$, respectively. In the same way, we can complete the proof according to the strong duality theorem. □

4. CASE STUDY

In this section, we examine our case study in Guangdong Province, China. Guangdong province has the highest number of typhoons landing in Mainland China, with 184 recorded typhoons hitting the province from 1949 to 2018. The typhoon data used in this paper comes from the CMA Tropical Cyclone Database (https://tcdata.typhoon.org.cn/), and the population data are from Guangdong Statistical Yearbook 2015. The case study network consists of 6 MDCs nodes and 88 PSAs nodes (each PSA is also a PD), which are created by using the publicly available geographic information system (GIS). Please see Figure 1 for details.

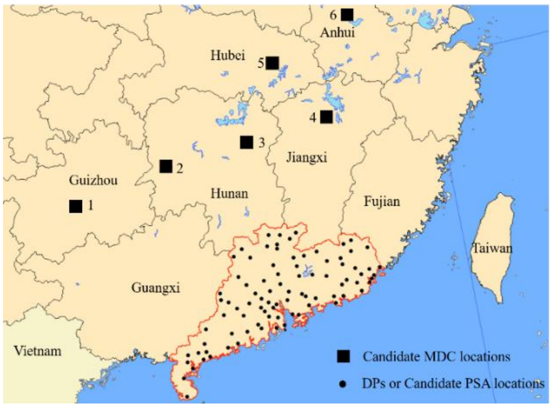

Figure 1. Case Study Network.

We obtain 3 main typhoon tracks based on historical data and consultation with meteorological experts. Each typhoon track has 5 categories. Therefore, 15 disaster scenarios are generated in this paper. We calculate the nominal probability of disaster scenarios based on historical typhoon data from 1949 to 2008. Other parameters such as fixed costs and transportation costs refer to Rawls and Turnquist (2010). Without loss of generality, $\hat{C}_{ik}^s$, $\hat{C}_{jk}^s$, $\hat{V}_j^s$, and $\hat{D}_k^s$ are set to $0.2\bar{C}_{ik}^s$, $0.2\bar{C}_{jk}^s$, $0.1\bar{V}_j^s$, and $0.1\bar{D}_k^s$, respectively. The uncertainty budgets $\Gamma^{s,C1}$, $\Gamma^{s,C2}$, $\Gamma^{s,V}$, and $\Gamma^{s,D}$ are set to 10, 20, 1, and 1, respectively. p_{low}^s and p_{up}^s are set to -0.05 and 0.05, respectively. The coverage parameter τ in K_j is equal to 120 (km). All cases are implemented using Python 3.6 and solved via Gurobi 9.0.2 on an Intel i7-7700 HQ with 8-Core processors and 32 GB of RAM.

4.1 Analysis of computational results

All the cases can be solved within 5 minutes, reflecting the high efficiency of the equivalent deterministic model proposed in this paper. We choose location 2 as MDC, where the number of emergency supplies in reserve is 4181.9 (ten thousand). The computational results of the selected PSAs are shown in Figure 2.

We further compare the 3DRO model with the stochastic optimization (SO) and RO models. The SO model does not consider the uncertainty of scenario probability, and the RO model only selects the worst-case scenario out of 15 scenarios. Based on the historical typhoon data from 1949 to 2008, we first calculated the solutions of the first stage of the 3DRO, SO, and RO models, respectively. After that, the typhoon data from 2009 to 2018 is used as a test set to evaluate the effects of different model solutions. The computational results are demonstrated in Table 1.

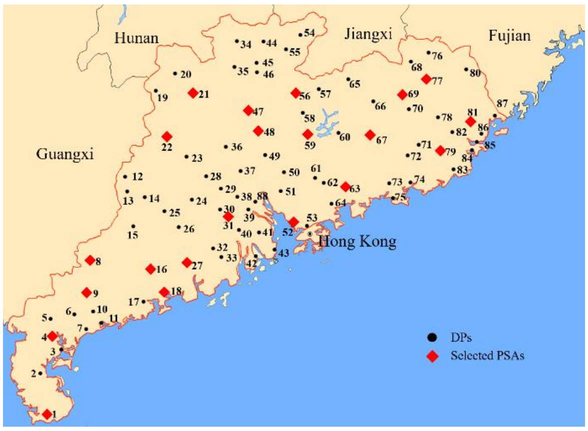

Figure 2. Candidate list of selected PSAs.

It can be seen from Table 1 that the cost of the first stage of the SO model is the least because it does not consider the uncertainty of the probability of disaster scenarios. However, it has the highest unsatisfied penalty in the test set. The reason may be that the SO model has the phenomenon of "optimizer's curse" in the case of small samples, that is, the actual performance of the solutions is worse than the theoretical result. Conversely, the RO model has the highest cost in the first stage because it focuses on the worst-case scenario. Although the unsatisfied penalty cost of the RO approach is 0, its solution is too conservative. The 3DRO model makes full use of the existing historical data and considers the uncertainty of the probability of disaster scenarios. Therefore, the total cost of the 3DRO model is the least.

Table 1 Computational results of different models

Costs (in $ millions)	3DRO	SO	RO
Fixed cost of MDCs	5.3	2.5	7.7
Fixed cost of PSAs	1.1	0.7	0.9
Relief supplies prepositioning cost	260.7	98.6	452.3
Unsatisfied penalty cost	0	434.1	0
Total cost	282.4	560.4	518.5

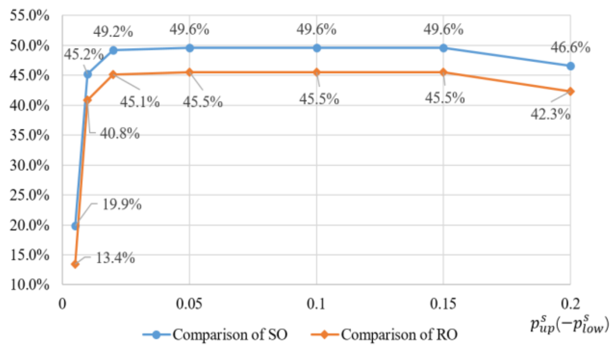

Figure 3. The improvement of 3DRO over SO and RO

The conservatism of the 3DRO model is related to parameters p_{low}^s and p_{up}^s, which can be determined by disaster managers. Therefore, we further conducted a sensitivity analysis for p_{low}^s

and p_{up}^{s}. The results are shown in Figure 3, which also reflects the superiority of the 3DRO model.

4.2 Comparison of the two-stage model

To illustrate the superiority of the 3-stage model proposed in this paper, we compare our model with the 2-stage DRO model consisting of stages 2 and 3. We randomly select locations to open MDCs. The set coverage model is used to determine the candidate list of PSAs to ensure that all DPs can be covered by PSAs. It is assumed that $z_i \sim U\,[2000, 5000]$. Repeat 100 trials to calculate the mean value. Table 2 illustrates the necessity of integrated decision-making for stages 1, 2, and 3. Compared to models that only consider two stages, our proposed three-stage model improved 24.5% on average.

Table2 Computational results of 3-stage DRO model Vs. 2-stage DRO model

Costs (in $ millions)	Stages of 1, 2, and 3	Stages of 2 and 3
Fixed cost of MDCs	5.3	4.5
Fixed cost of PSAs	1.1	0.7
Relief supplies prepositioning cost	260.7	213.3
Unsatisfied penalty cost	0	82.0
Total cost	282.4	374.0

5. CONCLUSIONS

This paper explores how to integrate the decisions of facility location, emergency supplies prepositioning, and distribution under predictable disasters. One of our major contributions is that we focus on the three stages of predictable disasters comprehensively and introduce a novel three-stage distributionally robust optimization (3DRO) in the context of predictable disasters. However, the 3DRO model we proposed is highly nonlinear. This paper presents a deterministic equivalent formulation of the 3DRO model to make it computationally tractable. A real case study in China is conducted to demonstrate the superiority of our proposed model by comparing it to stochastic optimization, robust optimization, and two-stage models. The computational results also show that: First, disaster managers should make full use of historical data knowledge and consider the uncertainty of knowledge when making decisions. Second, when making decisions of relief supplies prepositioning in the context of predictable disasters, it is essential to consider the three stages comprehensively.

This paper does not consider the impact of the deployment time of emergency supplies on the relief decisions. The reason is that the deployment time of supplies is usually determined in China. In the following research, we are interested in optimizing the deployment time of emergency supplies and developing efficient algorithms for solving the larger-scale instances.

ACKNOWLEDGEMENTS

This research is supported by the China Scholarship Council and Doctoral Dissertation Innovation Fund of Northwestern Polytechnical University under grant number CX2022059.

REFERENCES

Bertsimas, D., and Sim, M. (2004). The price of robustness. *Operations Research*, 52(1): 35-53.

Boyd, S., and Vandenberghe, L. (2004). Convex optimization, *Cambridge University Press*, New York.

Aleksandrova, M., Kaltenborn, M., Malerba, D., Radtke, K., and Strupat, C. (2021). World Risk Report 2021. *Bündnis Entwicklung Hilft, Law of Peace and Armed Conflict and Ruhr University Bochum*, Bochum.

Dalal, J., and Üster, H. (2021). Robust emergency relief supply planning for foreseen disasters under evacuation-side uncertainty. *Transportation Science*, 55(3): 791-813.

Liu, K., Li, Q., and Zhang, Z.H. (2019). Distributionally robust optimization of an emergency medical service station location and sizing problem with joint chance constraints. *Transportation Research Part B: Methodological*, 119: 79-101.

Ma, L., Liu, Y., and Liu, Y. (2020). Distributionally robust design for bicycle-sharing closed-loop supply chain network under risk-averse criterion. *Journal of Cleaner Production*, 246, 118967.

Ni, W., Shu, J., and Song, M. (2018). Location and emergency inventory pre-positioning for disaster response operations: Min-max robust model and a case study of Yushu earthquake. *Production and Operations Management*, 27(1):160-183.

Pacheco, G. G., and Batta, R. (2016). Forecast-driven model for prepositioning supplies in preparation for a foreseen hurricane. *Journal of the Operational Research Society*, 67(1): 98-113.

Paul, J. A., and Zhang, M. (2019). Supply location and transportation planning for hurricanes: A two-stage stochastic programming framework. *European Journal of Operational Research*, 274(1): 108-125.

Rawls, C. G., and Turnquist, M. A. (2010). Pre-positioning of emergency supplies for disaster response. *Transportation Research Part B: Methodological*, 44(4): 521-534.

Rezapour, S., Farahani, R. Z., and Morshedlou N. (2021). Impact of timing in post-warning prepositioning decisions on performance measures of disaster management: A real-life application. *European Journal of Operational Research*, 293(1): 312-335.

Stauffer, J. M., and Kumar, S. (2021). Impact of incorporating returns into pre-disaster deployments for rapid-onset predictable disasters. *Production and Operations Management*, 30(2): 451-474.

Wang, D., Yang, K., and Yang, L. (2021). Risk-averse two-stage distributionally robust optimisation for logistics planning in disaster relief management. *International Journal of Production Research*, doi: 10.1080/00207543.2021.2013559.

Wang, X. J., and Paul, J. A. (2020). Robust optimization for hurricane preparedness. *International Journal of Production Economics*, 221: 107464.